2.2.3 Consider F = { A → D, B → CE, E → H, D → E, E → C } Prove by Amstrong:

a) F |= B → H

E 🡪 H |= (t.t) CE 🡪 CH |= (f.r) CE 🡪 H (1)

B 🡪 CE (2) (gt).

(1), (2) |= (b.c) B 🡪 H ◼

b) F |= AB → CH

E 🡪 H |= (t.t) CE 🡪 CH (1)

B → CE (2)

(1), (2) |= B 🡪 CH |= (t.t) AB 🡪 ACH |= (f.r) AB 🡪 CH ◼

**Exercise 2.2.4**

Consider F = { D → BK, AB → GK, B → H, CE → AG, H → E, K → G, EH → K, G → AH }

Prove by Amstrong:

a) F |= AB → GH

b) F |= DE → AG

c) F |= BH → EK

a/ AB → GK |= (f.rã) AB → G (1)

B → H |= (t.t) AB → AH |= (f.rã) AB → H (2)

(1), (2) |= (h.) AB → GH ◼

b/ D → BK |= (t.t) DE → BEK (1)

K → G |= (t.t) BEK → GEK (2)

(1), (2) |= (b.c) DE → GEK |= (f.rã) DE → G (3).

D → BK |= (f.rã) D → K (4)

K → G (gt) (5)

(4), (5) |= (b.c) D → G (6)

G → AH (gt) (7)

(6), (7) |= (b.c) D → AH |= (f.rã) D → A |= (t.t) DE → AE |= (f.rã) DE → A (8).

(3), (8) |= (h.) DE → AG ◼

c/ B → H |= (t.t) BH → H (1)

H → E (2) (gt)

(1), (2) |= (b.c) BH → E (3).

B → H, H → E |= (b.c) B → E |= (t.t) BH → EH (4).

EH → K (gt) (5)

(4), (5) |= (b.c) BH → K (6)

(3), (6) |= (h.) BH → EK ◼